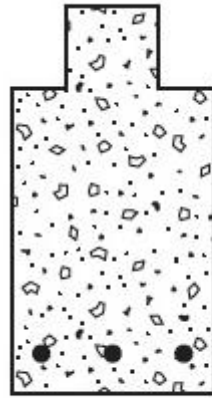
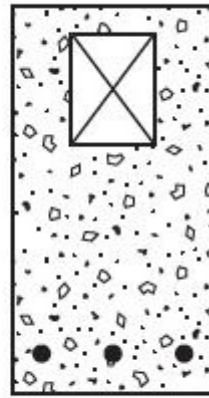


Analysis of Irregular Reinforced Concrete Beam

Sometimes a section different from the previously defined sections is needed for special requirements of structural members. For instance, sections such as those shown in figure below may be used in the precast concrete industry. The next examples explain the analysis of such sections.



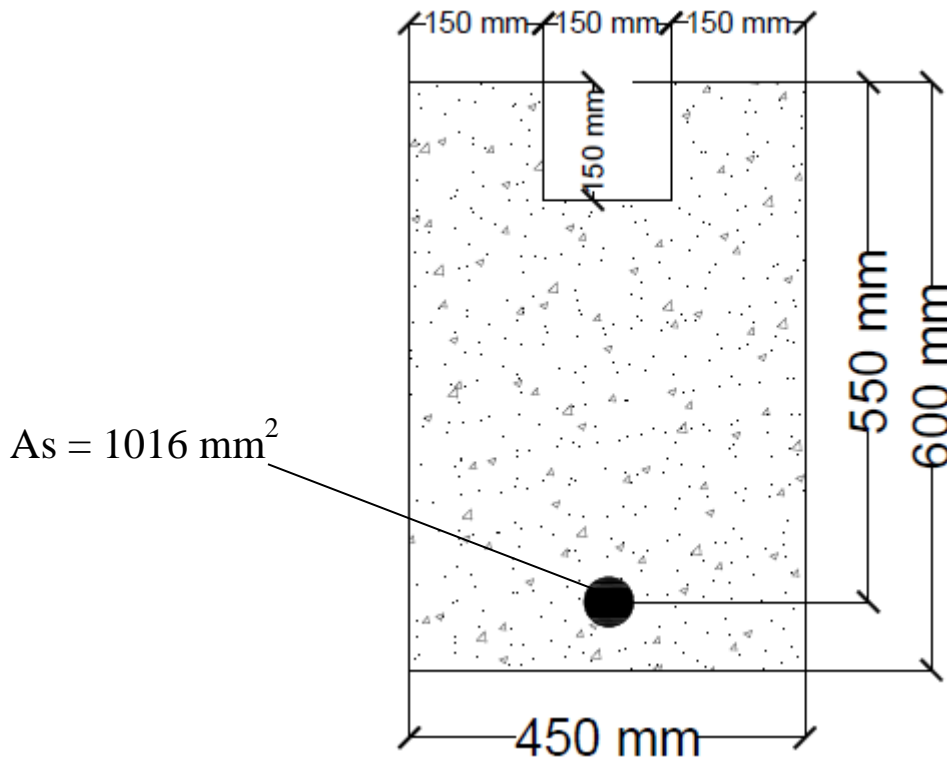
inverted T



beam with duct



Example 1: Check the adequacy of section shown below according to ACI-14 requirements and compute its design strength. $f'_c = 25$ MPa and $f_y = 400$ MPa



Solution

1. Check for A_{smax} and A_{smin} :

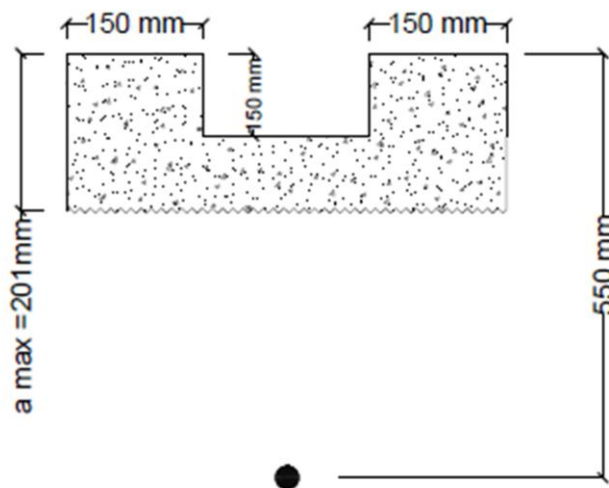
$$c_{max} = \frac{\epsilon_u}{\epsilon_u + 0.004} d \quad \text{where } \epsilon_u = 0.003$$

$$c_{max} = \frac{0.003}{0.003 + 0.004} d$$

$$c_{max} = 0.429d = 0.429 \times 550 = 236 \text{ mm}$$

$$\text{Find } a_{max} = \beta_1 c_{maximum} = 0.85 \times 236 = 201 \text{ mm}$$

$$\sum F_x = 0.85 \times 25 \times (2 \times 150 \times 150 + (201 - 150) \times 450) = A_{smax} \times 400$$



$$A_{s\max} = 3610 \text{ mm}^2 > A_s = 1016 \text{ mm}^2 \text{ O.K.}$$

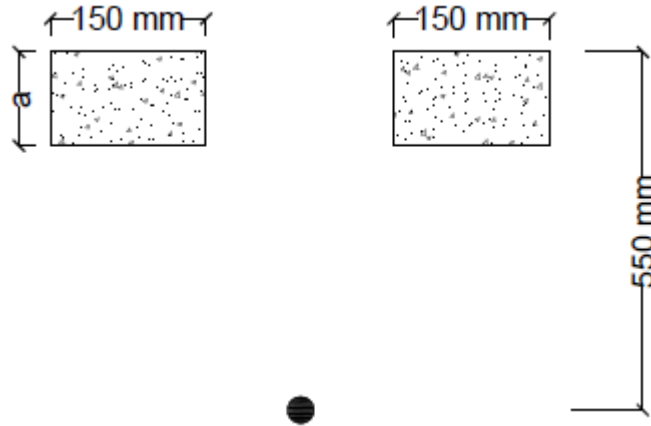
$$A_{s\min} = \frac{1.4}{f_y} bd = \frac{1.4}{400} \times 450 \times 550 = 866 \text{ mm}^2 < A_s \text{ O.K.}$$

2. Compute M_n :

Assume $a \leq 150 \text{ mm}$:

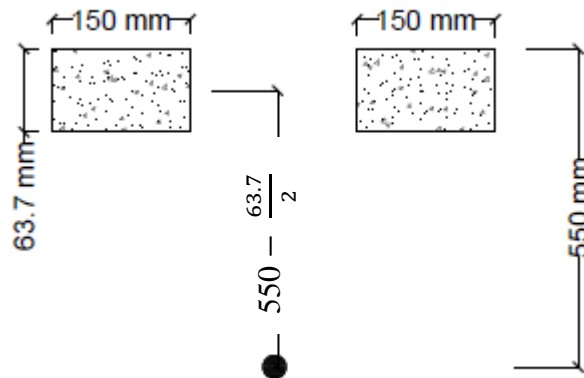
$$0.85 \times 25 \times 2 \times 150 \times a = 400 \times 1016$$

$$a = 63.7 \text{ mm} < 150 \text{ mm O.K.}$$



$$\sum M_{\text{about T}} = 0$$

$$M_n = 0.85 \times 25 \times 150 \times 2 \times 63.7 \times \left(550 - \frac{63.7}{2}\right) \times 10^{-6} = 210 \text{ kN.m}$$



3. Compute ϕ

$$a = 63.7 \text{ mm}$$

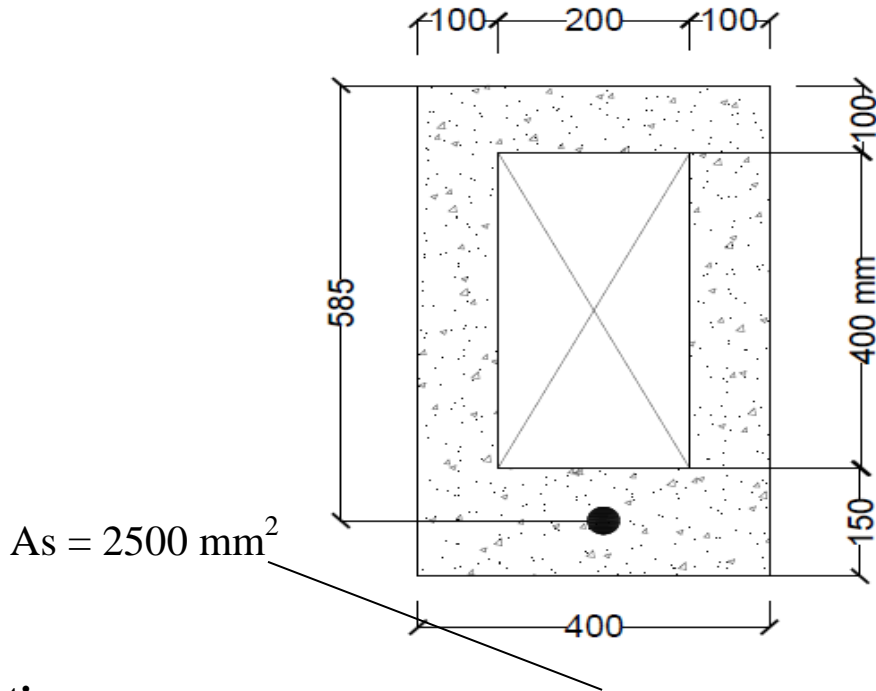
$$c = \frac{a}{\beta_1} = \frac{63.7}{0.85} = 74.9 \text{ mm}$$

$$\epsilon_t = \frac{d_t - c}{c} \epsilon_u = \frac{550 - 74.9}{74.9} \times 0.003 = 19 \times 10^{-3} > 0.005 \therefore \phi = 0.9$$

4. Compute ϕM_n

$$\phi M_n = 0.9 \times 210 = 189 \text{ kN.m} \blacksquare$$

Example 2: Check the adequacy of section for ACI-14 requirements and compute its design strength. $f'_c = 20$ MPa and $f_y = 400$ MPa



Solution

1. Check for A_{smax} and A_{smin} :

$$c_{max} = \frac{\epsilon_u}{\epsilon_u + 0.004} d \quad \text{where } \epsilon_u = 0.003$$

$$c_{max} = \frac{0.003}{0.003 + 0.004} d$$

$$c_{max} = 0.429d = 0.429 \times 585 = 251 \text{ mm}$$

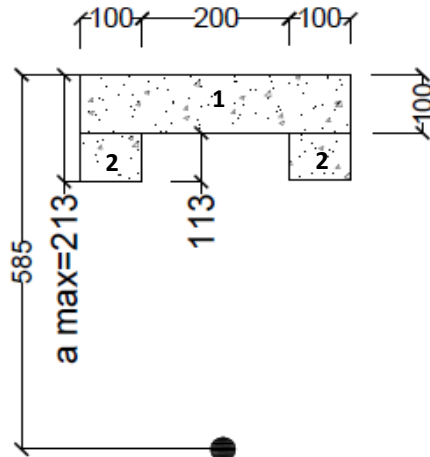
$$\text{Find } a_{max} = \beta_1 C_{maximum} = 0.85 \times 251 = 213 \text{ mm}$$

$$\sum F_x = 0$$

$$0.85 \times 20 \times \underbrace{(400 \times 100)}_{\text{Area Part 1}} + \underbrace{113 \times 100 \times 2}_{\text{Area Part 2}} = A_{smax} \times 400$$

$$A_{smax} = 2660.5 \text{ mm}^2 > A_s = 2500 \text{ mm}^2 \text{ O.K.}$$

$$A_{smin} = \frac{1.4}{f_y} bd = \frac{1.4}{400} \times 400 \times 585 = 819 \text{ mm}^2 < A_s \text{ O.K.}$$

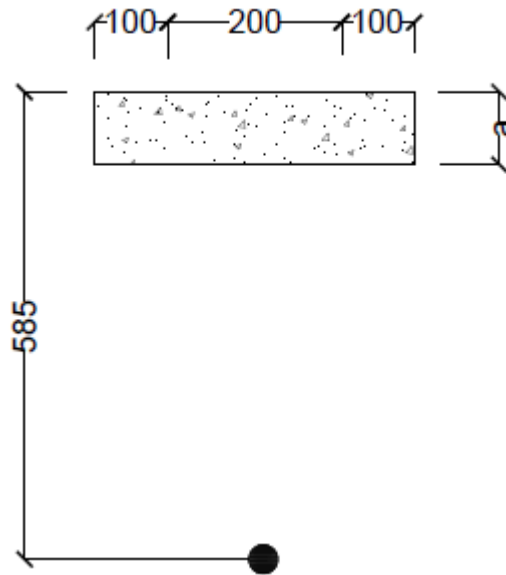


2. Compute M_n :

Assume $a \leq 100$ mm:

$$\sum F_x = 0$$

$$0.85 \times 20 \times 400 \times a = 400 \times 2500$$

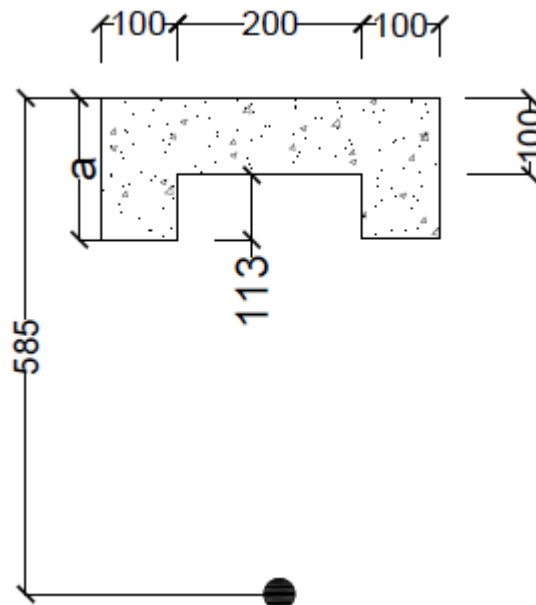


$a = 147$ mm > 100 mm not O.K.

$$\sum F_x = 0$$

$$0.85 \times 20 \times (200 \times 100 + 2 \times 100 \times a) = 400 \times 2500$$

$a = 194$ mm



$$\sum M_{\text{about T}} = 0$$

$$M_n = 0.85 \times 20 \times 200 \times 100 \times \left(585 - \frac{100}{2}\right) \times 10^{-6} + 0.85 \times 20 \times 100 \times 194 \times \left(585 - \frac{194}{2}\right) \times 10^{-6} = 503.7 \text{ kN.m}$$

3. Compute ϕ

$$a = 194 \text{ mm}$$

$$c = \frac{a}{\beta_1} = \frac{194}{0.85} = 228 \text{ mm}$$

$$\varepsilon_t = \frac{d_t - c}{c} \varepsilon_u = \frac{585 - 228}{228} \times 0.003 = 4.7 \times 10^{-3} < 0.005$$

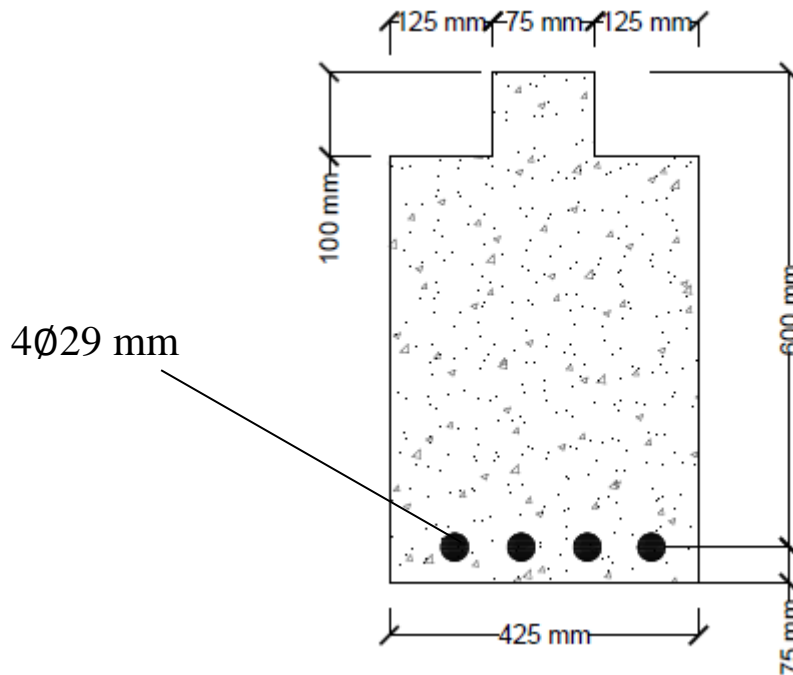
$$\therefore \phi = 0.483 + 83.3\varepsilon_t$$

$$\phi = 0.483 + 83.3 \times 4.7 \times 10^{-3} = 0.875$$

4. Compute ϕM_n

$$\phi M_n = 0.875 \times 503.7 = 440.7 \text{ kN.m} \blacksquare$$

Example 3: Check the adequacy of section for ACI-14 requirements and compute its design strength. $f'_c = 21$ MPa and $f_y = 420$ MPa



Solution

1. Check for A_{smax} and A_{smin} :

$$c_{max} = \frac{\epsilon_u}{\epsilon_u + 0.004} d \quad \text{where } \epsilon_u = 0.003$$

$$c_{max} = \frac{0.003}{0.003 + 0.004} d$$

$$c_{max} = 0.429d = 0.429 \times 600 = 257 \text{ mm}$$

$$\text{Find } a_{max} = \beta_1 C_{maximum} = 0.85 \times 257 = 218 \text{ mm}$$

$$\sum F_x = 0$$

$$0.85 \times 21 \times (175 \times 100 + 113 \times 425) = A_{smax} \times 400$$

$$A_{smax} = 2640 \text{ mm}^2 > A_s = \frac{\pi}{4} \times 29^2 = 2640 \text{ mm}^2 \text{ O.K.}$$

$$A_{smin} = \frac{1.4}{f_y} bd = \frac{1.4}{420} \times 425 \times 600 = 850 \text{ mm}^2 < A_s \text{ O.K.}$$

2. Compute M_n :

Assume $a \leq 100$ mm:

$$\sum F_x = 0$$

$$0.85 \times 21 \times 175 \times a = 4 \times 660 \times 420$$

$$a = 354 \text{ mm} > 100 \text{ mm not O.K.}$$

$$\sum F_x = 0$$

$$0.85 \times 21 \times (175 \times a + 2 \times (125 \times (a - 100))) = 4 \times 660 \times 420$$

$$a = 205 \text{ mm}$$

$$\sum M_{\text{about T}} = 0$$

$$M_n = 0.85 \times 21 \times (205 \times 175 \times 498 + 2(125 \times 105 \times 448)) \times 10^{-6}$$

$$M_n = 529 \text{ kN.m}$$

3. Compute ϕ

$$a = 205 \text{ mm}$$

$$c = \frac{a}{\beta_1} = \frac{205}{0.85} = 241 \text{ mm}$$

$$\varepsilon_t = \frac{d_t - c}{c} \varepsilon_u = \frac{600 - 241}{241} \times 0.003 = 4.47 \times 10^{-3} < 0.005$$

$$\therefore \phi = 0.483 + 83.3 \varepsilon_t$$

$$\phi = 0.483 + 83.3 \times 4.47 \times 10^{-3} = 0.855$$

4. Compute ϕM_n

$$\phi M_n = 0.855 \times 529 = 452 \text{ kN.m} \blacksquare$$

Example 4: Based on load share from slabs to beam have been determined, $W_u = 10 \text{ kN/m}$ (Including self-weight). Check the adequacy of section for ACI-14 requirements and compute its design strength. Beam span 5 m, $f'_c = 21 \text{ MPa}$ and $f_y = 420 \text{ MPa}$

